Lesson 9.5 Warm Up (Marker Boards)

1. Write an arithmetic series in summation notation for

5 + 8 + 11 + ... + 38

2. Simplify: $\dfrac{x^2 - 8x + 12}{x^2 - 11x + 30}$

Lesson 9.5 Geometric Series

Essential Understanding: Just as with finite arithmetic series, you can find the sum of a finite geometric series using a formula. You need to know the first term, the number of terms, and the common ratio.

A geometric series is the sum of the terms of a geometric sequence.

The sum S_n of a finite geometric series $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}$, $r \neq 1$, is

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex. What is the sum of the finite geometric series?

3 + 6 + 12 + 24 + ... + 3072

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex. What is the sum of the finite geometric series?

$$\sum_{n=0}^{20} 4\left(\frac{1}{2}\right)^n$$

Ex. What is the sum of the geometric sequence?

-15 + 30 - 60 + 120 - 240 + 480

Ex. What is the sum of the geometric series?

$$\sum_{n=1}^{10} 5 \cdot (-2)^{n-1}$$

Ex. What is the sum of the geometric series?

4 + 12 + 36 + ... + 2916

The Soldier's Reasonable Request A famous story involves a soldier who rescues his king in battle. The king grants him any prize "within reason" from the riches of the kingdom. The soldier asks for a chessboard with a single kernel of wheat on the first square, two kernels of wheat on the second square, then four, then eight, and so on for all 64 squares of the chessboard. The king decides that the request is reasonable.

According to the story, how many total kernels of wheat did the soldier request?

The Rest of the Story A bushel of wheat contains about a million kernels. The total US output of wheat in a recent year was just over 2.1 billion bushels. How many years of production at that level would it take the United States to produce enough wheat to satisfy the soldier's "reasonable" request?

Ex. To save money for a vacation, you set aside $100. For each month thereafter, you plan to set aside 10% more than the previous month. How much money will you save in 12 months?

The terms of a geometric series grow rapidly when the common ratio is greater than 1. Likewise, they diminish rapidly when the common ratio is between 0 and 1. In fact, they diminish so rapidly that an infinite geometric series has a finite sum.

Key Concept Infinite Geometric Series

An infinite geometric series with first term a_1 and common ratio $|r| < 1$ has a finite sum

$$S = \frac{a_1}{1 - r}.$$

An infinite geometric series with $|r| \geq 1$ does not have a finite sum.

To say that an infinite series has a sum means that the sequence of partial sums converges to a number S as n gets very large.

When an infinite series does not converge to a sum, the series diverges. An infinite geometric series with $|r| \geq 1$ diverges (thus, if $|r| < 1$ it converges).

Ex. Does the series converge or diverge? If it converges, what is the sum?

$$1 + \frac{1}{2} + \frac{1}{4} + \ldots$$

Ex. Does the series converge or diverge? If it converges, what is the sum?

$$\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)\left(-\frac{5}{4}\right)^n$$

Ex. Does the series converge or diverge? If it converges, what is the sum?

$$\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+\ \ldots$$

Ex. Does the series converge or diverge? If it converges, what is the sum?

$$\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\ \ldots$$