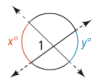
Lesson 12.4 Angle Measures & Segment Lengths

Essential Understanding: Angles formed by intersecting lines have a special relationship to the related arcs formed when the lines intersect a circle. In this lesson, you will study angles and arcs formed by lines intersecting either within a circle or outside a circle.

Theorem 12-13

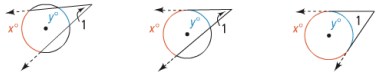
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(x + y)$$



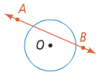
Theorem 12-14

The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.

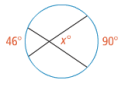


$$m\angle 1 = \frac{1}{2}(x - y)$$

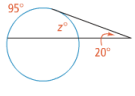
When a line intersects a circle at two different points, the line is called secant. In the diagram below, line AB is secant.



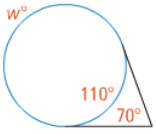
Ex. What is the value of x°?



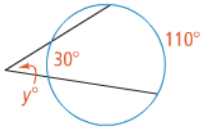
Ex. What is the value of z°?



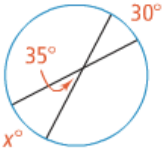
1 What is the value of w?



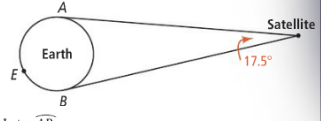
2 What is the value of y?



3 What is the value of x?

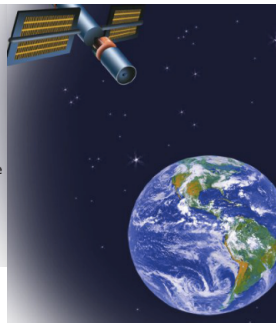


Satellite A satellite in a geostationary orbit above Earth's equator has a viewing angle of Earth formed by the two tangents to the equator. The viewing angle is about 17.5°. What is the measure of the arc of Earth that is viewed from the satellite?



Let $m\widehat{AB} = x$.

Then $m\widehat{AEB} = 360 - x$.



4 A departing space probe sends back a picture of Earth as it crosses Earth's equator. The angle formed by the two tangents to the equator is 20°. What is the measure of the arc of the equator that is visible to the space probe?

*Is the probe or the geostationary satellite in the previous problem closer to Earth? Explain.

In summary, when finding angle measures from secant and tangent lines, when the vertex is:

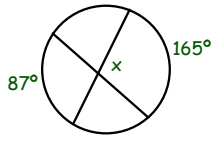
a. on = arc/2

b. in = add arcs / 2

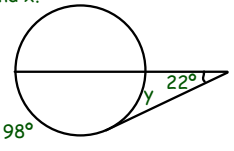
c. outside = subtract arcs / 2

Lesson 12.4 Day 2 Warm Up

1. Find x.



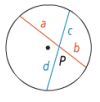
2. Find x.



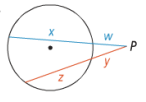
Essential Understanding There is a special relationship between two intersecting chords, two intersecting secants, or a secant that intersects a tangent. This relationship allows you to find the lengths of unknown segments.

Theorem 12-15

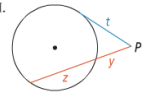
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.

I. 

$a \cdot b = c \cdot d$

II. 

$(w + x)w = (y + z)y$

III. 

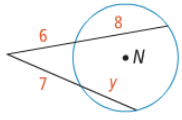
$(y + z)y = t^2$

Secant secant outside: whole * outside = whole * outside

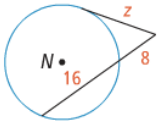
Secant secant inside: piece * piece = piece * piece

Secant tangent: whole * outside = tangent^2

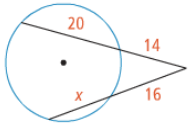
Ex. Find the value of y.



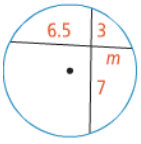
Ex. Find the value of z.



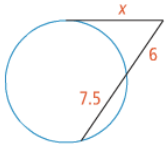
Ex. What is the value of x?



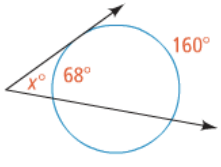
Ex. What is the value of m?



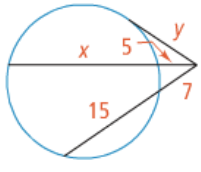
5 What is the value of x?



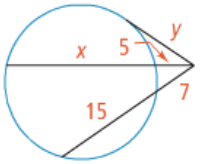
6 What is the value of x°?



7 What is the value of x? Round to the nearest tenth.



8 What is the value of y?



In summary, when finding segment lengths:

a. secant/secant inside = piece * piece = piece * piece

b. secant/secant outside = whole * outside = whole * outside

c. secant/tangent = whole * outside = tangent^2