Lesson 3.4 Warm Up (Clickers)

1. Solve the system: $3x - 4y = 8$

   $2x - 4y = -3$

2. A friend told you that she found the correlation coefficient of her line of best fit was 1.5. What is wrong with her statement?

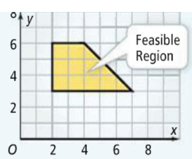
Lesson 3.4 Linear Programming

Essential Understanding: Some real-world problems involve multiple linear relationships. Linear programming accounts for all of these linear relationships and gives the solution to the problem.

Linear programming is a method for finding a minimum or maximum value of some quantity, given a set of constraints, or limits.

The constraints in a linear programming situation form a system of inequalities, like the one at the right. The graph of the system is the **feasible region**. It contains all the points that satisfy all the constraints.

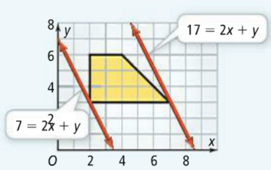
$$\begin{cases} x \geq 2 \\ y \geq 3 \\ y \leq 6 \\ x + y \leq 10 \end{cases}$$



The quantity you are trying to maximize or minimize is modeled with an **objective function**. Often this quantity is cost or profit. Suppose the objective function is $C = 2x + y$.

Graphs of the objective function for various values of C are parallel lines. Lines closer to the origin represent smaller values of C.

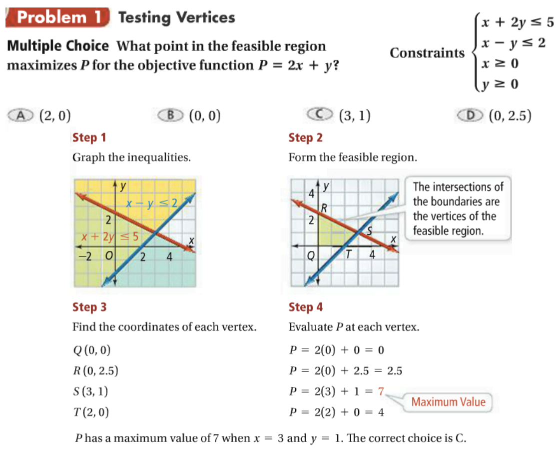
The graphs of the equations $7 = 2x + y$ and $17 = 2x + y$ intersect the feasible region at (2, 3) and (7, 3). These vertices of the feasible represent the least and the greatest values for the objective function.



Key Concept Vertex Principle of Linear Programming

If there is a maximum or a minimum value of the linear objective function, it occurs at one or more vertices of the feasible region.

You can solve a problem using linear programming by testing in the objective function all of the vertices of the feasible region.


Problem 1 Testing Vertices

Multiple Choice What point in the feasible region maximizes P for the objective function $P = 2x + y$?

Constraints $\begin{cases} x + 2y \le 5 \\ x - y \le 2 \\ x \ge 0 \\ y \ge 0 \end{cases}$

(A) (2, 0) (B) (0, 0) (C) (3, 1) (D) (0, 2.5)

Step 1
Graph the inequalities.

Step 2
Form the feasible region.

Step 3
Find the coordinates of each vertex.

$Q(0, 0)$
$R(0, 2.5)$
$S(3, 1)$
$T(2, 0)$

Step 4
Evaluate P at each vertex.

$P = 2(0) + 0 = 0$
$P = 2(0) + 2.5 = 2.5$
$P = 2(3) + 1 = 7$ Maximum Value
$P = 2(2) + 0 = 4$

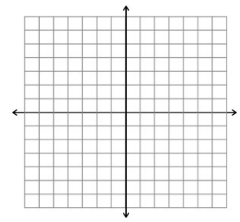
P has a maximum value of 7 when $x = 3$ and $y = 1$. The correct choice is C.

Ex. What point in the feasible region minimizes P for the objective function P = 5x - y?

Constraints: $x + y \le 1$

$4x + y \ge -2$

$x \le 0$



a. (-1, 2) b. (0, -2)

c. (0, 1) d. (-0.5, 0)

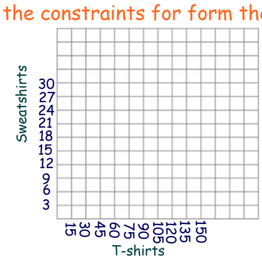
Ex. You are screen printing T-shirts and sweatshirts to sell at the Polk County Blues Festival and are working with the following constraints.

- You have at most 20 hours to make shirts
- You want to spend no more than \$600 on supplies.
- You want to have at least 50 items to sell.

How many T-shirts and how many sweatshirts should you make to maximize your profit? How much is the maximum profit?

Step 1: Write the constraints and the objective function.

Step 2: Graph the constraints for form the feasible region.



Step 3: Find the coordinates of each vertex.

Step 4: Evaluate P at each vertex. Recall P = 6x + 20y.

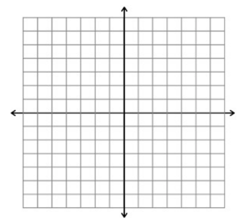
Ex. Graph each system of constraints. Then name the vertices of the feasible region.

$x \leq 5$

$y \leq 4$

$x \geq 0$

$y \geq 0$



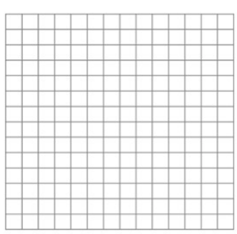
Ex. Graph each system of constraints. Name the vertices of the feasible region. Then find the values of x and y that maximize the objective function.

$x + y \leq 8$

$2x + y \leq 10$

$x \geq 0$

$y \geq 0$



Maximum for N = 100x + 40y