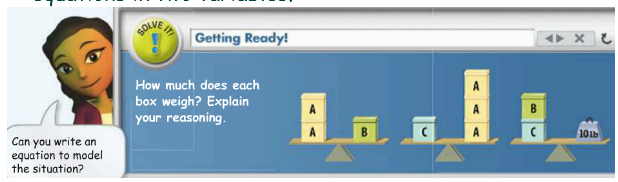
Lesson 3.5 Warm Up

1. Write the equation of the line in slope-intercept form that goes through the points (-1, 5) and (0, 2).

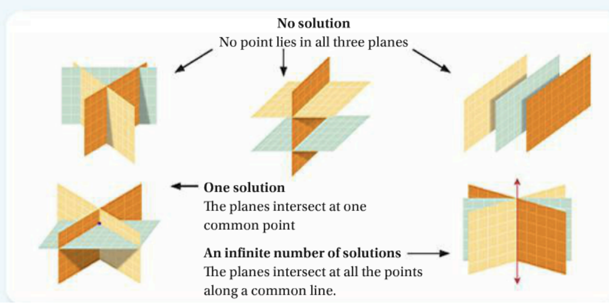
2. What is a residual?

Lesson 3.5 Systems with Three Variables

Essential Understanding: To solve systems of three equations in three variables you can use some of the same algebraic methods you used to solve systems of two equations in two variables.



You can represent systems of equations in three variables as graphs in three dimensions. The graph of an equation of the form $Ax + By + Cz = D$, where A, B, and C are not all zero, is a plane. You can show the solutions of a three-variable system graphically as the intersection of planes.



You can use elimination and substitution methods to solve a system of three equations in three variables by working with the equations in pairs. You will use one of the equations twice. When one point represents the solution of a system of equations in three variables, write it as an ordered triple (x, y, z).

Ex. What is the solution of the system?
$$\begin{cases} ① & 2x - y + z = 4 \\ ② & x + 3y - z = 11 \\ ③ & 4x + y - z = 14 \end{cases}$$

Step 1 Pair the equations to eliminate z. Then you will have two equations in x and y.

Add.

$$\begin{array}{lrcl} ① & 2x - y + z &=& 4 \\ ② & x + 3y - z &=& 11 \\ \hline ④ & 3x + 2y &=& 15 \end{array}$$

Subtract.

$$\begin{array}{lrcl} ② & x + 3y - z &=& 11 \\ ③ & 4x + y - z &=& 14 \\ \hline ⑤ & -3x + 2y &=& -3 \end{array}$$

Step 2 Write the two new equations as a system. Solve for x and y.

Add and solve for y.

$$\begin{array}{lrcl} ④ & 3x + 2y &=& 15 \\ ⑤ & -3x + 2y &=& -3 \\ \hline & 4y &=& 12 \\ & y &=& 3 \end{array}$$

Substitute $y = 3$ and solve for x.

$$\begin{array}{lrcl} ④ & 3x + 2y &=& 15 \\ & 3x + 2(3) &=& 15 \\ & 3x &=& 9 \\ & x &=& 3 \end{array}$$

Step 3 Solve for z. Substitute the values of x and y into one of the original equations.

$$\begin{array}{lrcll} ① & 2x - y + z &=& 4 & \text{Use equation ①.} \\ & 2(3) - 3 + z &=& 4 & \text{Substitute.} \\ & 6 - 3 + z &=& 4 & \text{Simplify.} \\ & z &=& 1 & \text{Solve for } z. \end{array}$$

Step 4 Write the solution as an ordered triple. The solution is $(3, 3, 1)$.

Ex. What is the solution of the system? Check your answer in all three original equations.

$$\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$$

1 What is the solution of the system? (write your answer as an ordered triple)

$3x + y - z = 1$

$x + 2y + z = 4$

$3x - y - z = 3$

Sometimes you may have to multiply an equation by a number so that variables cancel.

Ex. What is the solution of the system?

$$\begin{cases} x + y + 2z = 3 \\ 2x + y + 3z = 7 \\ -x - 2y + z = 10 \end{cases}$$

Another way to solve a system of three variables is to solve for one of the variables and then substitute it into the other two equations.

Ex. Solve the system: $\begin{cases} 2x + 3y - 2z = -1 \\ x + 5y = 9 \\ 4z - 5x = 4 \end{cases}$

Ex. Solve the system: $\begin{cases} x - 2y + z = -4 \\ -4x + y - 2z = 1 \\ 2x + 2y - z = 10 \end{cases}$

Ex. You manage a clothing store and budget \$6000 to restock 200 shirts. You can buy T-shirts for \$12 each, polo shirts for \$24 each, and rugby shirts for \$36 each. If you want to have twice as many rugby shirts as polo shirts, how many of each type of shirt should you buy?