Lesson 3.6 Solving Systems Using Matrices (Clickers)

Essential Understanding: You can use a matrix to represent and solve a system of equations without writing the variables.

A matrix is a rectangular array of numbers. You usually display the array within the brackets. The dimensions of a matrix are the numbers of rows and columns in the array.

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 6 & 5 & 3 \end{bmatrix}$$

3 columns

2 rows

Matrix A has 2 rows and 3 columns and is a 2×3 matrix, read "2 by 3." You can write it as A or $A_{2 \times 3}$.

Each number in a matrix is a **matrix element**. You can identify a matrix element by its row and column numbers. In matrix A, a_{12} is the element in row 1 and column 2. a_{12} is the element 4.

Ex. What is element a_{23} in matrix A?

$$A = \begin{bmatrix} 4 & -9 & 17 & 1 \\ 0 & 5 & 8 & 6 \\ -3 & -2 & 10 & 0 \end{bmatrix}$$

1 What is element a_{13} in matrix A?

$$A = \begin{bmatrix} 4 & -9 & 17 & 1 \\ 0 & 5 & 8 & 6 \\ -3 & -2 & 10 & 0 \end{bmatrix}$$

You can represent a system of equations efficiently with a matrix. Each matrix row represents an equation. The last matrix column shows the constants to the right of the equal signs. Each of the other columns shows the coefficients of one of the variables.

Ex. $x + 3y = 7$

$3x + y = -8$

$$\begin{array}{ccc} x\text{-coefficients} & y\text{-coefficients} & \text{constants} \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 7 \\ 3 & 1 & -8 \end{array}\right]$$

The 1's are coefficients of x and y.

Draw a vertical bar to replace the equal signs and separate the coefficients from the constants.

Ex. How can you represent the system of equations with a matrix?

$2x + y = 9$

$x - 6y = -1$

Ex. How can you represent the system of equations with a matrix?

$x - 3y + z = 6$

$x + 3z = 12$

$y = -5x + 1$

Ex. How can you represent the system of equations with a matrix?

$-4x - 2y = 7$

$3x + y = -5$

Ex. How can you represent the system of equations with a matrix?

$4x - y + 2x = 1$

$y + 5z = 20$

$2x = -y + 7$

Ex. What linear system of equations does this matrix represent?

$$\left[\begin{array}{cc|c} 5 & 2 & 7 \\ 0 & 1 & 9 \end{array}\right]$$

2 What linear system does the matrix represent? (separate your equations with a semicolon)

$$\left[\begin{array}{cc|c} 2 & 0 & 6 \\ 5 & -2 & 1 \end{array}\right]$$

You can use a matrix that represents a system of equations to solve the system. In this way, you do not have to write the variables. To solve the system using the matrix, use the steps for solving by elimination. Each step is a **row operation**.

Your goal is to use row operations to get a matrix in the form $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ or $\left[\begin{array}{ccc|} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

Notice that the first matrix represents the system $x = a, y = b$, which then will be the solution of a system of two equations in two unknowns. The second matrix represents the system $x = a, y = b$, and $z = c$.

Key Concept Row Operations

Switch any two rows.	$\begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 5 \end{bmatrix}$	becomes	$\begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 3 \end{bmatrix}$
Multiply a row by a constant.	$\begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 3 \end{bmatrix}$	becomes	$\begin{bmatrix} 3 & 2 & 5 \\ 2\cdot 2 & -1\cdot 2 & 3\cdot 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 4 & -2 & 6 \end{bmatrix}$
Add one row to another.	$\begin{bmatrix} 3 & 2 & 5 \\ 4 & -2 & 6 \end{bmatrix}$	becomes	$\begin{bmatrix} 3+4 & 2-2 & 5+6 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 11 \\ 4 & -2 & 6 \end{bmatrix}$

Combine any of these steps.

What is the solution of the system? $\begin{cases} x + 4y = -1 \\ 2x + 5y = 4 \end{cases}$

$\left[\begin{array}{cc|c} 1 & 4 & -1 \\ 2 & 5 & 4 \end{array}\right]$ Write the matrix for the system.

$$\begin{array}{rrrr} -2 & (1 & 4 & -1) \\ + & 2 & 5 & 4 \\ \hline & 0 & -3 & 6 \end{array}$$

Multiply Row 1 by -2. Add to Row 2. Replace Row 2 with the sum. Write the new matrix.

$\left[\begin{array}{cc|c} 1 & 4 & -1 \\ 0 & -3 & 6 \end{array}\right]$

$-\frac{1}{3}\ (0 \ \ -3 \ \ 6) = 0 \ \ 1 \ \ -2$

Multiply Row 2 by $-\frac{1}{3}$. Write the new matrix.

$\left[\begin{array}{cc|c} 1 & 4 & -1 \\ 0 & 1 & -2 \end{array}\right]$

$$\begin{array}{rrrr} & 1 & 4 & -1 \\ + -4 & (0 & 1 & -2) \\ \hline & 1 & 0 & 7 \end{array}$$

Multiply Row 2 by -4. Add to Row 1. Replace Row 1 with the sum. Write the new matrix.

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -2 \end{array}\right]$

The solution to the system is $(7, -2)$.

Matrices that represent the solution of a system are in reduced row echelon form (rref). Many calculators have a rref function for working with matrices. This function will do all the row operations for you. You can use rref to solve a system of equations.

Use your calculator to solve the system:

$$\begin{cases} 2a + 3b - c = 1 \\ -4a + 9b + 2c = 8 \\ -2a + 2c = 3 \end{cases}$$