Chapter 5 Polynomials & Polynomial Functions

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Lesson 5.1 Polynomial Functions (Clickers)

Essential Understanding: A polynomial function has distinguishing "behaviors". You can look at its algebraic form and know something about tis graph. You can look at its graph and know something about its algebraic form.

A monomial is a real number, a variable, or a product of a real number and one or more variables with whole-number exponents. The degree of a monomial in one variable is the exponent of the variable. A polynomial is a monomial or a sum of monomials. The degree of a polynomial in one variable is the greatest degree among its monomial terms.

Key Concept Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function $P(x)$ in standard form is

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

$$P(x) = 4x^3 + 3x^2 + 5x - 2$$

Cubic term | Quadratic term | Linear term | Constant term

Recall the table below from algebra 1. You name a polynomial according to its highest degree and number of terms.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	5	1	monomial
1	linear	$x + 4$	2	binomial
2	quadratic	$4x^2$	1	monomial
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4	quartic	$2x^4 + 5x^2$	2	binomial
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	polynomial of 4 terms

Ex. Write each polynomial in standard form. What is the classification of each polynomial by degree and number of terms?

a. $3x + 9x^2 + 5$

b. $4x - 6x^2 + x^4 + 10x^2 - 12$

1 Write the polynomial in standard form? Classify it according to its degree and number of terms. (Separate your answers with a comma)

$3x^3 - x + 5x^4$

2 Write the polynomial in standard form. Classify it according to its degree and number of terms. (separate answers with a comma)

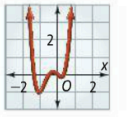
$2 - 3x$

The degree of a polynomial function affects the shape of its graph and determines the maximum number of turning points, or places where the graph changes direction. It also affects the end behavior, or the directions of the graph to the far left and to the far right.

The next slide shows you examples of polynomial functions and the four types of end behavior. The table also shows intervals where the functions are increasing and decreasing. A function is increasing when the y-values increase as x-values increase. A function is decreasing when the y-values decrease as x-values increase.

Key Concept Polynomial Functions

$y = 4x^4 + 6x^3 - x$

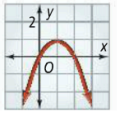


End Behavior: Up and Up

Turning Points: $(-1.07, -1.04)$, $(-0.27, 0.17)$, and $(0.22, -0.15)$

The function is decreasing when $x < -1.07$ and $-0.27 < x < 0.22$. The function increases when $-1.07 < x < -0.27$ and $x > 0.22$.

$y = -x^2 + 2x$



End Behavior: Down and Down

Turning Point: $(1, 1)$

The function is increasing when $x < 1$ and is decreasing when $x > 1$.

$y = x^3$

End Behavior: Down and Up

Zero turning points.

The function is increasing for all x.

$y = -x^3 + 2x$

End Behavior: Up and Down

Turning Points: $(-0.82, -1.09)$ and $(0.82, 1.09)$

The function is decreasing when $x < -0.82$ and when $x > 0.82$. The function is increasing when $-0.82 < x < 0.82$.

You can determine the end behavior of a polynomial function of degree n from the leading term ax^n of the standard form.

End Behavior of a Polynomial Function With Leading Term ax^n

	n Even ($n \neq 0$)	n Odd
a Positive	Up and Up	Down and Up
a Negative	Down and Down	Up and Down

Ex. Describe the end behavior of the polynomial

$f(x) = -2x^2 - 4x + 2$

Ex. What is the end behavior of the graph?

a. $y = 4x^3 - 3x$

b. $y = -2x^4 + 8x^3 - 2x + 4$

3 What is the end behavior of $y = -5x^3 + 2x^2 - 3$?

A up and up

B up and down

C down and down

D down and up

4 What is the end behavior of $y = 10x^6 - 3x^5 - 4x + 2$?

A up and up

B up and down

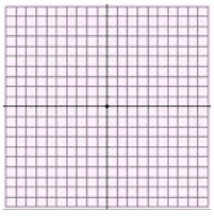
C down and down

D down and up

Ex. What is the graph of the cubic function? Describe the graph, including end behavior, turning points, and increasing/decreasing intervals.

$f(x) = 1/2x^3$

x	y
-2	
-1	
0	
1	
2	



Ex. What is the graph of the cubic function? Describe the graph, including end behavior, turning points, and increasing/decreasing intervals.

$f(x) = 3x - x^3$

x	y
-2	
-1	
0	
1	
2