Lesson 5.2 Warm Up (Clickers)

1. What is the end behavior of $y = -2x^5 - 4x + 3$?

2. Classify the polynomial according to its degree and number of terms: $-3x^2$.

3. What is the vertex of the quadratic equation:

   $y = -3x^2 - 12x + 3$?

Lesson 5.2 Polynomials, Linear Factors, and Zeros

Essential Understanding: Find the zeros of a polynomial function will help you factor the polynomial, graph the function, and solve the related polynomial equation.

Ex. What is the factored form of $x^3 - 2x^2 - 15x$?

Ex. What is the factored form of $x^3 - x^2 - 12x$?

Key Concepts   Roots, Zeros, and x-intercepts

The following are equivalent statements about a real number b and a polynomial $P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$.

- $x - b$ is a linear factor of the polynomial $P(x)$.
- b is a zero of the polynomial function $y = P(x)$.
- b is a root (or solution) of the polynomial equation $P(x) = 0$.
- b is an x-intercept of the graph of $y = P(x)$.

Ex. What are the zeros of $y = (x + 2)(x - 1)(x - 3)$?

   Graph the function.
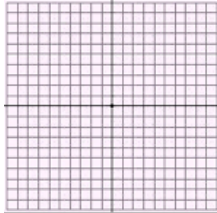
Ex. What are the zeros of $x(x - 3)(x + 5)$?

   Graph the function.
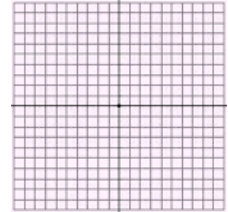
Theorem Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Ex. What is a cubic polynomial function in standard form with zeros -2, 2, and 3?

1 What is a quadratic polynomial function with zeros 3 and –3?

2 What is a cubic polynomial function with zeros 3, 3, and –3?

When there are multiple zeros with the same value, it is called a zero with multiplicity. For example, if there are two zeros with value of 3, you would say 3 is a zero with multiplicity of 2. **In general, a is a zero of multiplicity n means that x - a appears n times as a factor.**

If the graph of a polynomial function has several turning points, the function can have a **relative maximum** and a **relative minimum**. A relative maximum is the value of the function at an up-to-down turning point. A relative minimum is the value of the function at a down-to-up turning point.
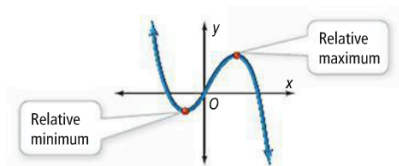
Use a graphing calculator to find the relative maximum and minimum values.

$$f(x) = x^3 + 3x^2 - 24x$$

3 What are the relative maximum and minimum of
$f(x) = 3x^3 + x^2 - 5x$?
(Round answers to the nearest hundredth)

Ex. The design of a digital box camera maximizes the volume while keeping the sum of the dimensions at 6 inches. If the length must be 1.5 times the height, what should each dimension be?

Ex. A metalworker wants to make an open box from a sheet of metal, but cutting equal squares from each corner as shown.

   a. Write an expression for the length, width, and height of the open box.
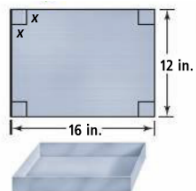
   b. Write a function for the volume.

c. Find the maximum volume of the box and the side length of the cut-out squares that generates this volume.