Lesson 5.7 The Binomial Theorem

Essential Understanding: You can use a pattern of coefficients and the pattern $a^n, a^{n-1}b, a^{n-2}b^2, \ldots, a^2b^{n-2}, ab^{n-1}, b^n$ to write the expansion of $(a + b)^n$.

To expand the power of a binomial in general, first multiply as needed. Then write the polynomial in standard form.

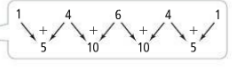
Consider the expansions of $(a + b)^n$ for the first few values of n:

Row	Power	Expanded Form	Coefficients Only
0	$(a + b)^0$	1	1
1	$(a + b)^1$	$1a^1 + 1b^1$	1 1
2	$(a + b)^2$	$1a^2 + 2a^1b^1 + 1b^2$	1 2 1
3	$(a + b)^3$	$1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$	1 3 3 1
4	$(a + b)^4$	$1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$	1 4 6 4 1

The coefficients only column matches the numbers in Pascal's Triangle. Pascal's Triangle is a triangular array of numbers in which the first and last number of each row is 1. Each of the other numbers in the row is teh sum of the two numbers above it.

For example, to generate row 5, use the sums of the adjacent elements in the row above it.

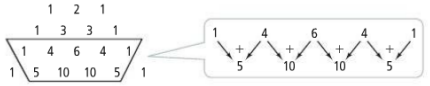
Row	Pascal's Triangle
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1



Ex. What is the expansion of $(a + b)^6$?

For example, to generate row 5, use the sums of the adjacent elements in the row above it.

Row Pascal's Triangle
0 1
1 1 1
2 1 2 1
3 1 3 3 1
4 1 4 6 4 1
5 1 5 10 10 5 1
6 1 6 15 20 15 6 1
7 1 7 21 35 35 21 7 1
8 1 8 28 56 70 56 28 8 1



Ex. What is the expansion of $(a + b)^8$?

1 What is the expansion of $(a+b)^3$?

The Binomial Theorem gives a general formula for expanding a binomial.

Take note

Theorem Binomial Theorem

For every positive integer n,

$$(a+b)^n = P_0a^n + P_1a^{n-1}b + P_2a^{n-2}b^2 + \cdots + P_{n-1}ab^{n-1} + P_nb^n$$

where $P_0, P_1, \ldots, P_n$ are the numbers in the nth row of Pascal's Triangle.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Ex. Expand $(3x - 2)^5$. *Notice the binomial is a difference. Change 3x - 2 to 3x + -2 and expand from there.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Ex. Expand $(2x - 3)^4$.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Expand $(x + 7)^3$

2 Expand

$(4x+2)^3$

3 Expand

$(x-3)^4$