

Chapter 8 Rational Functions

- 8-1 Inverse Variation
- 8-2 The Reciprocal Function Family
- 8-3 Rational Functions and Their Graphs
- 8-4 Rational Expressions
- 8-5 Adding and Subtracting Rational Expressions
- 8-6 Solving Rational Equations

Lesson 8.1 Inverse Variation

Essential Understanding: If a product is constant, where the constant is positive, a decrease in the value of one factor must accompany an increase in the value of the other factor.

As an equation, direct variation has the form $y = kx$, where $k \neq 0$, inverse variation can have the form $xy = k$, $y = k/x$, or $x = k/y$, $k \neq 0$. When two quantities vary inversely, as one quantity increases, the other decreases proportionally. For both inverse and direct variation, k is the constant of variation.

Ex. Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

Think

How can you tell if the quantities vary directly or inversely? If the product of corresponding x - and y -values is constant, they vary inversely. If the ratio of corresponding x - and y -values is constant, they vary directly.

x	y
2	15
4	7.5
10	3
15	2

As x increases, y decreases. This might be an inverse relationship. A plot confirms that an inverse relationship is possible. Test to see whether xy is constant.

$$\begin{array}{ll} 2 \cdot 15 = 30 & 4 \cdot 7.5 = 30 \\ 10 \cdot 3 = 30 & 15 \cdot 2 = 30 \end{array}$$

The product of each pair is 30, so $xy = 30$ and y varies inversely with x . The constant of variation is 30 and the function model is $y = \frac{30}{x}$.

Ex. Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

x	y
2	10
4	8
10	3
15	1.5

Ex. Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

x	y
0.2	8
0.5	20
1.0	40
1.5	60

1 Ex. Is the relationship between the variables a direct variation, an inverse variation, or neither?

A direct

B inverse

C neither

x	y
0.2	40
0.5	16
1.0	8.0
2.0	4.0

2 Ex. Is the relationship between the variables a direct variation, an inverse variation, or neither?

A direct

B inverse

C neither

x	y
0.5	40
1.2	12
2	10
2.5	6

Ex. Suppose x and y vary inversely, and $x = 4$ when $y = 12$.

a. What function models the inverse variation?

b. Graph this function.

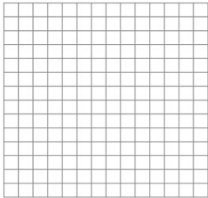


c. What is y when $x = 10$?

Ex. Suppose x and y vary inversely, and $x = 9$ when $y = -7$.

a. What is the function that models the inverse variation?

b. Graph this function.



c. What is y when $x = 2$?

3 Suppose x and y vary inversely, and $x = 2$ when $y = 8$.
What is the function of the inverse variation?

Ex. Your math class has decided to pick up litter each weekend in a local park. Each week there is approximately the same amount of litter. The table shows the number of students who worked each of the first four weeks of the project and the time needed for the pickup. What function models the data?

Park Cleanup Project				
Number of students (n)	3	5	12	17
Time in minutes (t)	85	51	21	15

How many students should there be to complete the project in at most 30 minutes each week?

When one quantity varies with respect to two or more quantities, you have a combined variation. when one quantity varies directly with two or more quantities, you have joint variation. The volume of a cone varies jointly with the area of the base and the height of the cone. $V = k Bh$.

Key Concept Combined Variations	
Combined Variation	Equation Form
z varies jointly with x and y .	$z = kxy$
z varies jointly with x and y and inversely with w .	$z = \frac{kxy}{w}$
z varies directly with x and inversely with the product wy .	$z = \frac{kx}{wy}$

Ex. The number of bags of grass seed n needed to reseed a yard varies directly with the area a to be seeded and inversely with the weight w of a bag of seed. If it takes two 3-lb bags to seed an area of 3600 ft^2 , how many 3-lb bags will seed 9000 ft^2 ?

Ex. The number of bags of mulch you need to cover a planting area varies jointly with the area to be mulched a in square feet and the depth of the mulch d in feet. If you need 10 bags to mulch 120 ft^2 to a depth of 3 in., how many bags do you need to mulch 200 ft^2 to a depth of 4 in.?