Lesson 8.3 Warm Up (Marker Boards)

1. Given (-2, 3) and (5, y) and that the points come from an inverse variation equation, what is the value of y?

2. Locate the asymptotes and state the domain and range for the function: $y = \dfrac{3}{x - 2} + 5$

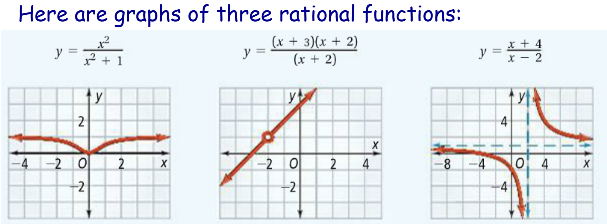
Lesson 8.3 Rational Functions and Their Graphs

Essential Understanding: If a function has a polynomial in its denominator, its graph has a gap at each zero of the polynomial. The gap could be a one-point hole in the graph, or it could be the location of a vertical asymptote for the graph.

A rational function is a function thta you can write in teh form $f(x) = \dfrac{P(x)}{Q(x)}$ where P(x) and Q(x) are polynomials.

The domain of f(x) is all real numbers except those values for which Q(x) = 0.

Here are graphs of three rational functions:



There is no value of x such that the denominator is equal to zero. Thus the graph is continuous.

Since the graph is undefined at x = -2, this graph is discontinuous.

Key Concept Point of Discontinuity

If a is a real number for which the denominator of a rational function $f(x)$ is zero, then a is not in the domain of $f(x)$. The graph of $f(x)$ is not continuous at $x = a$ and the function has a **point of discontinuity** at $x = a$.

The graph of $y = \frac{(x + 3)(x + 2)}{x + 2}$ has a **removable discontinuity** at $x = -2$. The hole in the graph is called a removable discontinuity because you could make the function continuous by redefining it at $x = -2$ so that $f(-2) = 1$.

The graph of $y = \frac{x + 4}{x - 2}$ has a **non-removable discontinuity** at $x = 2$. There is no way to redefine the function at 2 to make the function continuous.

When you are looking for discontinuities, it is helpful to factor the numerator and denominator as a first step. The factors of the denominator will reveal the points of discontinuity. The discontinuity caused by $(x - a)^n$ in the denominator is removable if the numerator also has $(x - a)^n$ as a factor.

Ex. What are the domain and points of discontinuity of each rational function? Are the points of discontinuity removable or non-removable? What are the x- and y-intercepts? $y = \frac{x + 3}{x^2 - 4x + 3}$

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Ex. What are the domain and points of discontinuity of each rational function? Are the points of discontinuity removable or non-removable? What are the x- and y-intercepts? $y = \frac{x^2 - 1}{x^2 + 3}$

Key Concept Vertical Asymptotes of Rational Functions

The graph of the rational function $f(x) = \frac{P(x)}{Q(x)}$ has a vertical asymptote at each real zero of $Q(x)$ if $P(x)$ and $Q(x)$ have no common zeros. If $P(x)$ and $Q(x)$ have $(x - a)^m$ and $(x - a)^n$ as factors, respectively and $m < n$, then $f(x)$ also has a vertical asymptote at $x = a$.

Ex. What are the vertical asymptotes for the graph of

$$y = \frac{(x + 1)}{(x - 2)(x - 3)} ?$$

Ex. What are the vertical asymptotes for the function

$$y = \frac{x^2 - 1}{x + 1} ?$$

Ex. What are the vertical asymptotes of the function

$$y = \frac{(x - 2)}{x^2 + 2x - 3} ?$$

Lesson 8.3 Continued Warm Up

1. Find the point(s) of discontinuity.

$$y = \frac{x + 5}{x^2 + 9x + 20}$$

2. Find the vertical asymptotes of the graph of the function $y = \frac{x - 3}{x^2 + 5x + 6}$

3. Determine whether the function has removable or non-removable discontinuity. $y = \frac{2x^2 + 5}{x^2 - 2x}$

While the graph of a rational function can have any number of vertical asymptotes, it can have no more than one horizontal asymptote.

Key Concept Horizontal Asymptote of a Rational Function

To find the horizontal asymptote of the graph of a rational function, compare the degree of the numerator m to the degree of the denominator n.

If $m < n$, the graph has horizontal asymptote $y = 0$ (the x-axis).

If $m > n$, the graph has no horizontal asymptote.

If $m = n$, the graph has horizontal asymptote $y = \frac{a}{b}$ where a is the coefficient of the term of greatest degree in the numerator and b is the coefficient of the term of greatest degree in the denominator.

Ex. What is the horizontal asymptote for the rational function? $y = \dfrac{2x}{x - 3}$

Ex. What is the horizontal asymptote for the rational function? $y = \dfrac{x - 2}{x^2 - 2x - 3}$

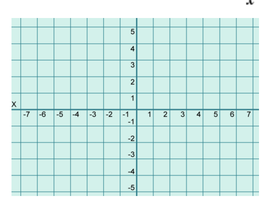
Ex. What is the horizontal asymptote for the rational function? $y = \dfrac{x^2}{2x - 5}$

Ex. What is the horizontal asymptote for the rational function? $y = \dfrac{-2x + 6}{x - 5}$

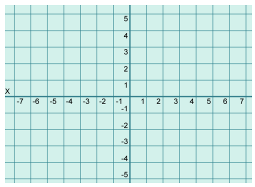
Ex. What is the horizontal asymptote for the rational function? $\dfrac{x - 1}{x^2 + 4x + 4}$

<u>Essential Understanding</u>: You can get a reasonable graph for a rational function by finding all intercepts and asymptotes. Sometimes you will also have to plot a few extra points to get a good sense of the shape of the graph.

Ex. What is the graph of the rational function $y = \dfrac{x^2 + x - 12}{x^2 - 4}$?



Ex. What is the graph of $y = \dfrac{x+3}{x^2 - 6x + 5}$?



Ex. What is the graph of the function $y = \dfrac{4x}{x^3 - 4x}$?