

Chapter 9 Transformations

Lesson 9.1 Translations & Lesson 9.2 Reflections

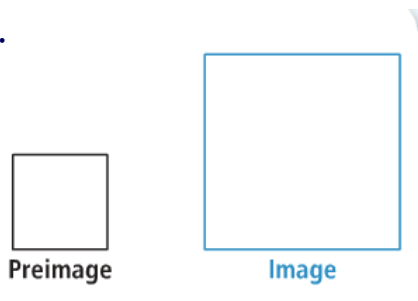
Essential Understanding: You can change the position of a geometric figure so that the angle measures and the distance between any two points of a figure stay the same.

A transformation of a geometric figure is a function, or mapping that results in a change in the position, shape, or size of the figure.

In a transformation, the original figure is the preimage. The resulting figure is the image. Some transformations preserve distance and angle measures. To preserve distance means that the distance between any two points of the image is the same as the distance between the corresponding points of the preimage. To preserve angles means that the angles of the image have the same angle measure as the corresponding angles of the preimage. A transformation that preserves distance and angle measures is called a rigid motion.

Are the following rigid transformations?

A.



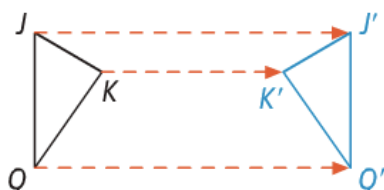
B.



C.



A transformation maps every point of a figure onto its image and may be described with arrow notation (\rightarrow). **Prime notation** (') is sometimes used to identify image points. In the diagram below, K' is the image of K .



$\triangle JKQ \rightarrow \triangle J'K'Q'$
 $\triangle JKQ$ maps onto $\triangle J'K'Q'$.

Notice that you list corresponding points of the preimage and image in the same order, as you do for corresponding points of congruent or similar figures.

Take Note

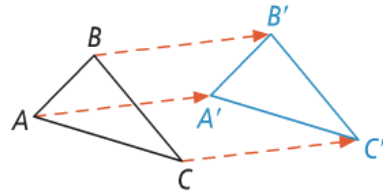
Key Concept Translation

A **translation** is a transformation that maps all points of a figure the same distance in the same direction.

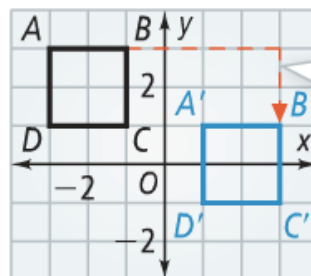
You write the translation that maps $\triangle ABC$ onto $\triangle A'B'C'$ as $T(\triangle ABC) = \triangle A'B'C'$. A translation is a rigid motion with the following properties.

If $T(\triangle ABC) = \triangle A'B'C'$, then

- $AA' = BB' = CC'$
- $AB = A'B', BC = B'C', AC = A'C'$
- $m\angle A = m\angle A', m\angle B = m\angle B', m\angle C = m\angle C'$

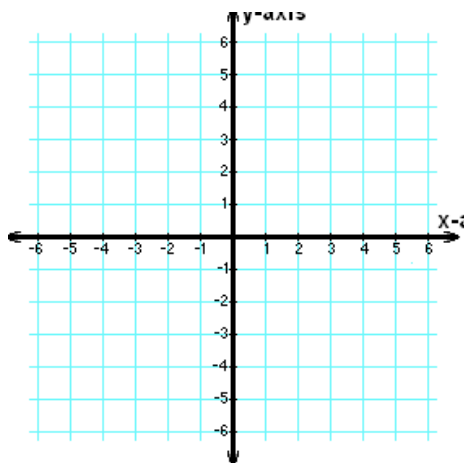
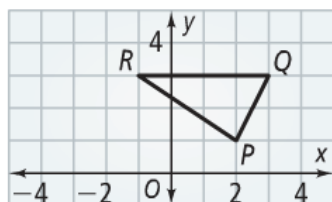


Below is an example of a translation that moved 4 units right and 2 units down. To show this, the following notation is used: $T_{\langle 4, -2 \rangle}(ABCD) = A'B'C'D'$

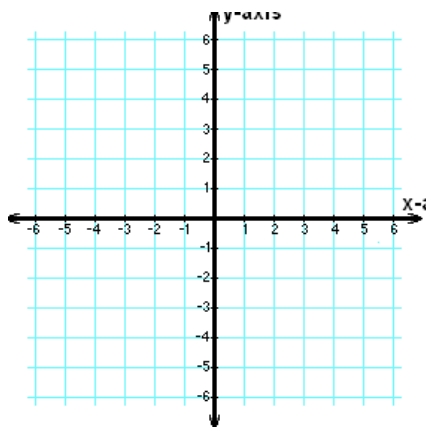
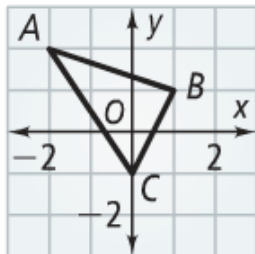


B moves 4 units right and 2 units down.

Ex. What are the vertices of $T_{\langle -2, -5 \rangle}(\triangle PQR)$? Graph the image of $\triangle PQR$.

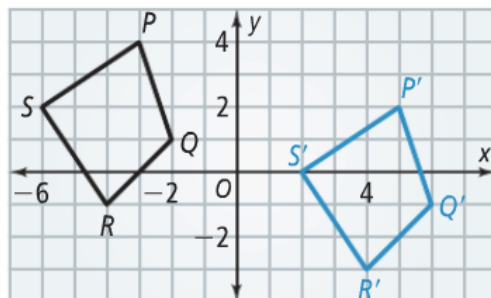


Ex. What are the vertices of $T_{\langle 1, -4 \rangle}(\triangle ABC)$? Copy $\triangle ABC$ and graph its image.



Reasoning Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What relationships exist among these three segments? How do you know?

Ex. What is a rule that describes the translation that maps $PQRS$ onto $P'Q'R'S'$?



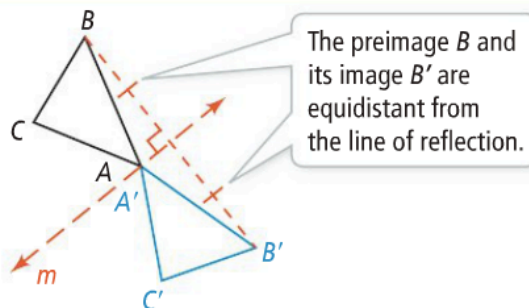
Essential Understanding: When you reflect a figure across a line, each point of the figure maps to another point the same distance from the line but on the other side. The orientation of the figure reverses.

Key Concept Reflection Across a Line

A **reflection** across a line m , called the **line of reflection**, is a transformation with the following properties:

- If a point A is on line m , then the image of A is itself (that is, $A' = A$).
- If a point B is not on line m , then m is the perpendicular bisector of $\overline{BB'}$.

You write the reflection across m that takes P to P' as $R_m(P) = P'$.



You can use the equation of a line of reflection in the function notation. For example, $R_{y=x}$ describes the reflection across the line $y = x$.

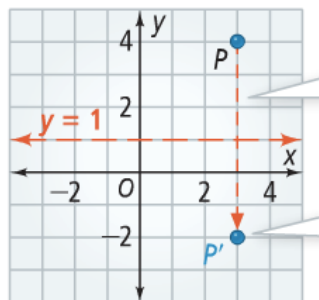
Multiple Choice Point P has coordinates $(3, 4)$. What are the coordinates of $R_{y=1}(P)$?

(A) $(3, -4)$

(B) $(0, 4)$

(C) $(3, -2)$

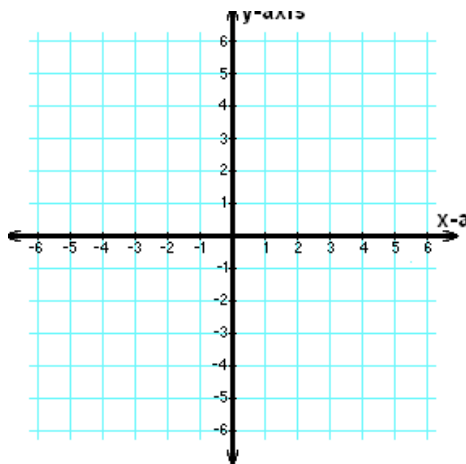
(D) $(-3, -2)$



Ex. $R_{x=1}(P) = P'$. What are the coordinates of P' ?

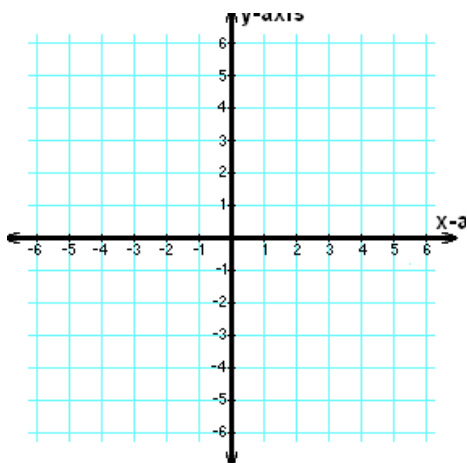
Note: Reflections are rigid transformations. What does this mean?

Coordinate Geometry Graph points $A(-3, 4)$, $B(0, 1)$, and $C(4, 2)$. Graph and label $R_{y\text{-axis}}(\triangle ABC)$.

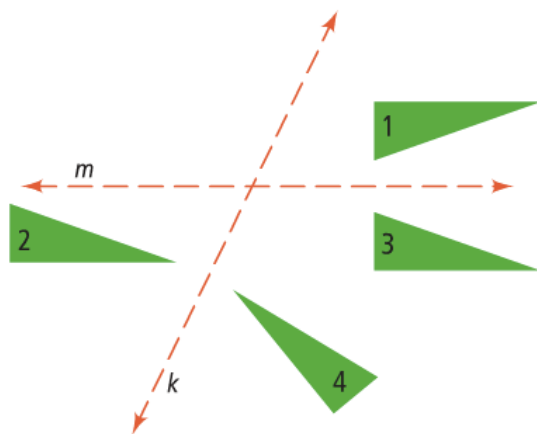


Ex. Graph $\triangle ABC$ from Problem 2. Graph and label $R_{x\text{-axis}}(\triangle ABC)$.

$A(-3, 4)$, $B(0, 1)$, and $C(4, 2)$.



Ex. Each triangle in the diagram is a reflection of another triangle across one of the given lines. How can you describe Triangle 2 by using a reflection rule?



Triangle 2 = $R_k(\text{Triangle 4})$

Ex. How can you use a reflection rule to describe Triangle 1? Explain.

