## Lesson 9.5 Arithmetic Series (Marker Boards)

**Essential Understanding:** When you know two terms and the number of terms in a finite arithmetic sequence, you can find the sum of the terms.

A series is the indicated sum of the terms of a sequence. A finite series, like a finite sequence, has a first term and a last term, while and infinite series continues without end.

| Finite Sequence: | Finite Series: |
|---|---|
| 6, 9, 12, 15, 18 | 6 + 9 + 12 + 15 + 18 (=60) |
| Infinite Sequence: | Infinite Series: |
| 3, 7, 11, 15, ... | 3 + 7 + 11 + 15 + ... |

An arithmetic series is a series whose terms form an arithmetic sequence (as shown above). When a series has a finite number of terms, you can use a formula involving the first and last term to evaluate the sum.

Sum of a Finite Arithmetic Series: $S_n = n/2(a_1 + a_n)$

where $a_1$ is the first term,

$a_n$ is the last term, and

n is the number of terms.

Ex. What is the sum of the even integers from 2 to 100?

Ex. What is the sum of the arithmetic series

4 + 9 + 14 + 19 + 24 + .... + 99?

1  What is the sum of the finite arithmetic series 14 + 17 + 20 + 23 + ...+ 116?

A company pays a $10,000 bonus to salespeople at the end of their first 50 weeks if they make 10 sales in their first week, and then improve their sales numbers by two each week thereafter. One salesperson qualified for the bonus with the minimum possible number of sales. How many sales did the salesperson make in week 50? In all 50 weeks?

You can use the Greek capital letter sigma, Σ, to indicate a sum. With it, you use limits to indicate how many terms you are adding. Limits are the least and greatest values of n in the series. You write the limits below and above the Σ to indicate the first and last terms of the series.

For example, you can write the series $3^2 + 4^2 + 5^2 + \cdots + 108^2$ as $\sum_{n=3}^{108} n^2$.

Upper limit: the series ends with $n = 108$.

The explicit formula for each term is $n^2$.

$$\sum_{n=3}^{108} n^2$$

Lower limit: the series begins with $n = 3$.

For an infinite series, summation notation shows ∞ as the upper limit.

To find the number of terms in a series written in Σ form, subtract the lower limit from the upper limit and add 1.

Ex. The number of terms in the series below is

108 – 3 + 1 = 106.

$$\sum_{n=3}^{108} n^2$$

Ex. What is summation notation for the series?

7 + 11 + 15 + ...+ 203 + 207

Ⓐ $\sum_{n=1}^{51}(4n + 3)$   Ⓑ $\sum_{n=1}^{50}(4n + 3)$   Ⓒ $\sum_{n=1}^{50}(7n)$   Ⓓ $\sum_{n=1}^{51}(7n)$

Ex. What is the summation notation for the series

-5 + 2 + 9 + 16 + ... + 268

Ex. What is summation notation for the series

500 + 490 + 480 + ... + 10?

Evaluate each summation below:

a. $\sum_{n=1}^{7}(n - 1)^2$

b. $\sum_{n=1}^{4}n^3$

Ex. Find the summation of $\displaystyle\sum_{n=1}^{40}(3n-8)$ ?

Obviously there is a shorter way to be able to evaluate summations. Follow the commands below to evaluate summations:

1. LIST-MATH-SUM

2. LIST-OPS-SEQ

3. EXPLICIT FORMULA-LOWER LIMIT-UPPER LIMIT

Ex. Find the summation of $\displaystyle\sum_{n=1}^{70}(5n+3)$